Remarks

2. Remark. The product can of course be calculated using cup product in $H^{\cdot}(M, \mathbb{Q})$.

3. Remark. It is likely that one can avoid the contrivance of defining $t(\mathcal{F})$ if one works in the cohomology algebra of an ambient variety, using the equation $t(N)ch(\Lambda^{\cdot}\mathcal{N}) = c_n(N)$. The properties of t being multiplicative and invertible simplify proofs but seem not to be essential. Also it may be more natural to define a compactly supported Chow ring such as the subring of the Chow ring of a compactified vector bundle generated by cycles not contained in the boundary. That is, the projection formula and the rule $x \cdot M = i_* i^* x$ directly imply Riemann Roch.

4. Remark. The Grothendieck ring $G(M)$ has a filtration by ideals I_c being generated by cycles of codimension c or larger. The Chow ring is the associated graded ring $A = \oplus_{c=0}^{n} I_c/I_{c+1}$ The Chern character homomorhism $G(M) \to A \otimes \mathbb{Q}$ identifies a lattice in $A \otimes \mathbb{Q}$ with a factor ring of $G(M)$ modulo an ideal. It is tempting not to use the Chern character, but to lift the proof into $G(M)$ by removing the symbol $ch(\quad)$ wherever it occurs in the proof, thus interpreting the Riemann Roch theorem as a theorem about $G(R)$. The fundamental class $c_n(\pi^* N)$ correspondingly must be replaced by an element $M \in G(R)$ such that $x \cdot M = i_* i^* x$, for instance an element whose chern *character* is equal to $c_n(\pi^* N)$.

5. Remark. It would of course be better to have a theorem which gives the dimension of $|D|$ even when D is not very positive. This amounts to looking at the locally free resolutions more carefully.

6. Remark. Just formally, there is nearly an identity in the Grothendieck ring $S^{\cdot}\mathcal{N} \cdot \lambda^{\cdot}\mathcal{N} = 1$ where odd degree terms of $\Lambda^{\cdot}\mathcal{F}$ count negatively. The first term is $\pi_* \pi^* \mathcal{N}$. The reason this is not quite right is that it represents an infinite sum of modules, and $G(R)$ describes only finitely-generated (coherent) sheaves. On the level of Chern characters one has a non-convergent series in the first term. It is though tempting to wonder whether a type of argument involving harmonic analysis might replace the Koszul resolution arguments. There would be nothing gained if one stayed in the 'virtual' world of Grothendieck groups, yet for a calculation of the dimension of $|D|$ for D not very positive this does make a difference.

7. Remark. The necessary involvement of log and exponential functions in the proof (see Remark 4 above) may suggest that arguments involving logarithmic forms might be more direct somehow. It is worthy checking whether anything there can lead to a proof not requiring relations in $G(M)$ which would work better for non highly positive D. Note however that the cohomology of logarithmic p forms on the compactified bundle amounts to *non* compactly supported cohomology of N, which is nothing but cohomology of M.

8. Remark The proof is very suggestive of arguments involving principal parts.....